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Nonabelian interactions from Hamiltonian BRST cohomology

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Abstract. Consistent Hamiltonian couplings between a set of vector fields and a system of matter fields are derived by means of BRST cohomological techniques.

1 Introduction

The cohomological approach to the Lagrangian BRST symmetry [1–5] stimulated the incorporation of new aspects within the cohomological BRST setting, like, for instance, a treatment of consistent interactions among fields with gauge freedom with the preservation of the number of gauge symmetries [6–10] from the perspective of the deformation of the solution to the master equation [11] with the help of the local BRST cohomology [12–16]. This procedure was proved to be an efficient deformation technique for many models of interest, like Chern–Simons models, Yang–Mills theories, the Chapline–Manton model, p-forms and chiral p-forms, Einstein's gravity theory, four- and eleven-dimensional supergravity, or BF models [11,17–32].

In the meantime, the Hamiltonian version of the BRST formalism [5,33–37] presents many useful and attractive features, like the implementation of the BRST symmetry in quantum mechanics [5] (Chapter 14), examination of anomalies [38], computation of local BRST cohomology [39], and also the explanation of the relationship with canonical quantization methods [40]. Recently, the Hamiltonian BRST setting has been enriched with the topic of constructing consistent interactions in gauge theories by means of the deformation technique and local cohomologies [41–44].

In this paper we investigate the consistent Hamiltonian interactions that can be introduced between a set of vector fields and a system of matter fields with the help of cohomological BRST arguments combined with the deformation technique. This approach represents an extension of our former results exposed in [45] related to the abelian case. Our method goes as follows. We begin with a "free" action written as the sum between the action for a set of vector fields and an action describing a matter theory, and construct the corresponding Hamiltonian BRST symmetry s, that simply decomposes into $s = \delta + \gamma$, with δ the Koszul–Tate differential and γ the exterior derivative along the gauge orbits. Its non-trivial action is essentially due to the first-class constraints involving the vector fields. It has been shown in [41–44] that the Hamiltonian problem of introducing consistent interactions in gauge theories can be reformulated as a deformation problem of the BRST charge and BRST-invariant Hamiltonian of a starting "free" theory. Following this line, we first compute the deformed BRST charge. This necessitates the (co)homological spaces $H(\gamma)$ and $H\left(\delta|\tilde{\mathrm{d}}\right)$, where $\tilde{\mathrm{d}} = \mathrm{d}x^i\partial_i$ represents the spatial part of the exterior space-time derivative. Based on these (co)homologies we obtain the result that the deformed BRST charge can be taken to be non-vanishing only at order one in the coupling constant. The consistency of the first-order deformation requires that the deformed first-class constraints form a Lie algebra in the Poisson (Dirac) bracket. Secondly, we investigate the equations responsible for the deformation of the BRST-invariant Hamiltonian. The first-order deformation equation reveals two different types of couplings. One involves only the vector fields and their momenta, and requires no further assumptions. The other demands that the matter theory should display some conserved Hamiltonian currents, equal in number to the number of vector fields. Consequently, it follows that the second type of couplings (between vector and matter fields) is of the form $j^\mu_a A^a_\mu$, where the j^μ_a denote the above mentioned conserved Hamiltonian currents. The equation that governs the second-order deformation of the BRST-invariant Hamiltonian definitely outputs the spatial part of the quartic vertex of pure Yang–Mills theory, and eventually other couplings involving both vector and matter fields. The appearance of the last type of couplings depends on the behaviour of the conserved currents under the gauge transformations generated by the deformed first-class constraints. Thus, if the spatial part of these currents, j^i_a, transform according to the adjoint representation of the Lie gauge algebra, then there are no second-order couplings between vector and matter fields, and, meanwhile, all types of three- and higher-order deformations can be taken to vanish. In the opposite case, at least the second-

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order deformation implying vector and matter fields is non-trivial, but in principle there might be other relevant higher-order interactions as well.

This paper is organized in seven sections. Section 2 briefly formulates the analysis of consistent Hamiltonian interactions that can be added to a "free" theory without changing its number of gauge symmetries as a deformation problem of the corresponding BRST charge and BRST-invariant Hamiltonian, finally expressed in terms of the so-called main equations. In Sect. 3 we determine the "free" Hamiltonian BRST differential. Based on this, in Sects. 4 and 5 we derive the deformed BRST charge, respectively, the deformed BRST-invariant Hamiltonian by means of cohomological techniques. In Sect. 6 we apply our procedure to two cases of interest, where the role of the matter fields is played by a set of scalar fields, respectively, by a collection of Dirac fields. Section 7 ends the paper with some conclusions.

2 Main Hamiltonian deformation equations

We assume a "free" Lagrangian theory subject to some gauge transformations. All the information on its Lagrangian gauge structure is contained in the solution to the master equation. It has been shown that the deformation of this solution leads to consistent interactions among fields with gauge freedom [5]. In the framework of the Hamiltonian setting, the structure of a given gauge theory is entirely determined by two quantities: the BRST charge and the BRST-invariant Hamiltonian. Similar to the Lagrangian deformation procedure, we can then reformulate the problem of constructing consistent Hamiltonian interactions in terms of the deformation of both the BRST charge and the BRST-invariant Hamiltonian.

As long as the interactions can be constructed in a consistent manner, the BRST charge of a given "free" theory, Ω_0, can be deformed as

$$\Omega_0 \rightarrow \Omega = \Omega_0 + g \int \mathrm{d}^{D-1}x\, \omega_1 + g^2 \int \mathrm{d}^{D-1}x\, \omega_2 + O\left(g^3\right)$$
$$= \Omega_0 + g\Omega_1 + g^2\Omega_2 + O\left(g^3\right), \tag{1}$$

where Ω satisfies the equation

$$[\Omega, \Omega] = 0, \tag{2}$$

and the symbol $[,]$ means either the Poisson or the Dirac bracket. (If the starting theory is purely first-class, we work with the Poisson bracket; if second-class constraints are also present, then we eliminate them, and use the Dirac bracket instead.) By projecting (2) on various powers in the deformation parameter (coupling constant) g, we arrive at the tower of equations

$$[\Omega_0, \Omega_0] = 0, \tag{3}$$
$$2\,[\Omega_0, \Omega_1] = 0, \tag{4}$$
$$2\,[\Omega_0, \Omega_2] + [\Omega_1, \Omega_1] = 0, \tag{5}$$
$$\vdots$$

Equation (3) is satisfied by assumption, while the resolution of the remaining equations in terms of the "free" BRST differential leads to the pieces $(\Omega_k)_{k>0}$. With the deformed BRST charge at hand, we deform the BRST-invariant Hamiltonian of the "free" theory, $H_{0\mathrm{B}}$, like

$$H_{0\mathrm{B}} \rightarrow H_{\mathrm{B}}$$
$$= H_{0\mathrm{B}} + g \int \mathrm{d}^{D-1}x\, h_1 + g^2 \int \mathrm{d}^{D-1}x\, h_2 + O\left(g^3\right)$$
$$= H_{0\mathrm{B}} + gH_1 + g^2 H_2 + O\left(g^3\right), \tag{6}$$

and demand that it obeys the relation

$$[H_{\mathrm{B}}, \Omega] = 0, \tag{7}$$

which implements that H_{B} is indeed the BRST-invariant Hamiltonian of the deformed system. Equation (7) can also be investigated order by order in the deformation parameter g, giving

$$[H_{0\mathrm{B}}, \Omega_0] = 0, \tag{8}$$
$$[H_{0\mathrm{B}}, \Omega_1] + [H_1, \Omega_0] = 0, \tag{9}$$
$$[H_{0\mathrm{B}}, \Omega_2] + [H_1, \Omega_1] + [H_2, \Omega_0] = 0, \tag{10}$$
$$\vdots$$

Equation (8) is again satisfied by hypothesis, while the others yield the components $(H_k)_{k>0}$. Equations (3)–(5), etc., and (8)–(10), etc., govern the Hamiltonian BRST deformation treatment, and will be called in the sequel the main equations.

3 Free BRST differential

We begin with a "free" action written as the sum between the action for a set of vector fields and an action describing a matter theory. We assume that the matter fields possess no gauge invariances of their own. The Hamiltonian canonical variables are denoted by $\left(A^a_\mu, \pi^\mu_a, y^{\alpha_0}\right)$, where $\left(A^a_\mu, \pi^\mu_a\right)$ correspond to the vector fields, while the y^{α_0} describe the matter theory. The non-vanishing fundamental Poisson (Dirac) brackets are taken in the form

$$\left[A^a_\mu, \pi^\nu_b\right] = \delta_\mu{}^\nu \delta^a{}_b, \quad \left[y^{\alpha_0}, y^{\beta_0}\right] = \omega^{\alpha_0\beta_0}, \tag{11}$$

with $\omega^{\alpha_0\beta_0}$ an invertible matrix (the distributional character was omitted for simplicity's sake). Due to the presence of the vector fields, the system is subject to the irreducible first-class constraints

$$G_{1a} \equiv \pi^0_a \approx 0, \quad G_{2a} \equiv -\partial_i \pi^i_a \approx 0, \tag{12}$$

and is endowed with the first-class Hamiltonian

$$H_0 = \int \mathrm{d}^{D-1}x \Bigg(\frac{1}{2}\pi_{ia}\pi^a_i + \frac{1}{4}F^a_{ij}F^{ij}_a - A^a_0 \partial_i \pi^i_a$$
$$+ \bar{h}_0\left(y^{\alpha_0}, \partial_i y^{\alpha_0}\right) \Bigg), \tag{13}$$

where $\bar{H}_0 = \int \mathrm{d}^{D-1}x\ \bar{h}_0\left(y^{\alpha_0}, \partial_i y^{\alpha_0}\right)$ represents the canonical Hamiltonian of the purely matter theory. The BRST charge of this system is given by

$$\Omega_0 = \int \mathrm{d}^{D-1}x \left(\pi_a^0 \eta_1^a - \left(\partial_i \pi_a^i\right) \eta_2^a\right), \tag{14}$$

the accompanying BRST-invariant Hamiltonian being

$$H_{0\mathrm{B}} = H_0 + \int \mathrm{d}^{D-1}x\ \eta_1^a \mathcal{P}_{2a}. \tag{15}$$

In (14) and (15) (η_1^a, η_2^a) are the fermionic ghost number one Hamiltonian ghosts, while $(\mathcal{P}_{1a}, \mathcal{P}_{2a})$ stand for the associated antighosts. The BRST complex is graded by the ghost number (gh), defined like the difference between the pure ghost number (pgh) and the antighost number (antigh), with

$$\mathrm{pgh}\left(A_\mu^a\right) = \mathrm{pgh}\left(\pi_a^\mu\right) = \mathrm{pgh}\left(y^{\alpha_0}\right) = 0, \tag{16}$$

$$\mathrm{pgh}\left(\eta_1^a\right) = \mathrm{pgh}\left(\eta_2^a\right) = 1,$$

$$\mathrm{pgh}\left(\mathcal{P}_{1a}\right) = \mathrm{pgh}\left(\mathcal{P}_{2a}\right) = 0, \tag{17}$$

$$\mathrm{antigh}\left(A_\mu^a\right) = \mathrm{antigh}\left(\pi_a^\mu\right) = \mathrm{antigh}\left(y^{\alpha_0}\right) = 0, \tag{18}$$

$$\mathrm{antigh}\left(\eta_1^a\right) = \mathrm{antigh}\left(\eta_2^a\right) = 0,$$

$$\mathrm{antigh}\left(\mathcal{P}_{1a}\right) = \mathrm{antigh}\left(\mathcal{P}_{2a}\right) = 1. \tag{19}$$

The "free" BRST symmetry $s\cdot = [\cdot, \Omega_0]$ splits as

$$s = \delta + \gamma, \tag{20}$$

where δ is the Koszul–Tate differential, graded according to the antighost number ($\mathrm{antigh}(\delta) = -1$, $\mathrm{antigh}(\gamma) = 0$), and γ is the exterior longitudinal derivative along the gauge orbits, graded in terms of the pure ghost number ($\mathrm{pgh}(\gamma) = 1$, $\mathrm{pgh}(\delta) = 0$). These operators act on the variables from the BRST complex via the definitions

$$\delta A_\mu^a = 0, \quad \delta \pi_a^\mu = 0, \quad \delta y^{\alpha_0} = 0, \tag{21}$$

$$\delta \eta_1^a = \delta \eta_2^a = 0, \quad \delta \mathcal{P}_{1a} = -\pi_a^0, \quad \delta \mathcal{P}_{2a} = \partial_i \pi_a^i, \tag{22}$$

$$\gamma A_0^a = \eta_1^a, \quad \gamma A_i^a = \partial_i \eta_2^a, \quad \gamma \pi_a^\mu = 0, \quad \gamma y^{\alpha_0} = 0, \tag{23}$$

$$\gamma \eta_1^a = \gamma \eta_2^a = 0, \quad \gamma \mathcal{P}_{1a} = \gamma \mathcal{P}_{2a} = 0, \tag{24}$$

that will be used in the sequel during the deformation process.

4 Deformation of the BRST charge

In this section we analyse the main equations (4) and (5), etc., that describe the deformation of the "free" BRST charge. Equation (4) written in a local form becomes

$$s\omega_1 = \partial_i k^i, \tag{25}$$

for some local k^i. In order to solve (25), we develop ω_1 according to the antighost number

$$\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1 + \cdots + \overset{(J)}{\omega}_1, \tag{26}$$

with

$$\mathrm{antigh}\left(\overset{(I)}{\omega}_1\right) = I, \quad \mathrm{pgh}\left(\overset{(I)}{\omega}_1\right) = 1, \tag{27}$$

where the last term in (26) can be assumed to be annihilated by γ, $\gamma \overset{(J)}{\omega}_1 = 0$. Thus, we need to know the cohomology of γ, $H(\gamma)$, in order to output $\overset{(J)}{\omega}_1$. Looking at the definitions (23) and (24), we find the result that $H(\gamma)$ is generated by F_{ij}^a, π_a^μ, y^{α_0}, $\mathcal{P}_{1a}$, $\mathcal{P}_{2a}$ and their spatial derivatives, as well as by the undifferentiated ghosts η_2^a. The ghosts η_1^a do not enter the cohomology of γ as they are γ-exact by virtue of the former definitions in (23). As a consequence, the general solution to the equation $\gamma\alpha = 0$ can be represented (up to a trivial term) by

$$\alpha = \alpha_M\left(\left[F_{ij}^a\right], \left[\pi_a^\mu\right], \left[y^{\alpha_0}\right], \left[\mathcal{P}_{1a}\right], \left[\mathcal{P}_{2a}\right]\right) e^M\left(\eta_2^a\right), \tag{28}$$

with $e^M\left(\eta_2^a\right)$ a basis in the finite-dimensional space of polynomials in the ghosts η_2^a, while the notation $f[q]$ signifies that f depends on q and its derivatives up to a finite order. Then, the equation $\gamma \overset{(J)}{\omega}_1 = 0$ possesses the solution

$$\overset{(J)}{\omega}_1 = \tilde{\omega}_J\left(\left[F_{ij}^a\right], \left[\pi_a^\mu\right], \left[y^{\alpha_0}\right], \left[\mathcal{P}_{1a}\right], \left[\mathcal{P}_{2a}\right]\right) e^{J+1}\left(\eta_2^a\right), \tag{29}$$

where $\mathrm{pgh}\left(e^{J+1}\left(\eta_2^a\right)\right) = J + 1$ and $\mathrm{antigh}\left(\tilde{\omega}_J\right) = J$. Related to the component of antighost number $(J-1)$, (25) becomes

$$\delta \overset{(J)}{\omega}_1 + \gamma \overset{(J-1)}{\omega}_1 = \partial_i m^i. \tag{30}$$

For the last equation to display solutions it is necessary that $\tilde{\omega}_J$ belongs to $H_J\left(\delta|\tilde{\mathrm{d}}\right)$. However, using the general results from [15,16] adapted to our situation, we have

$$H_J\left(\delta|\tilde{\mathrm{d}}\right) = 0, \quad \text{for } J > 1. \tag{31}$$

This implies that we can assume that the development (26) stops after the first two terms

$$\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1, \tag{32}$$

with $\gamma \overset{(1)}{\omega}_1 = 0$. Due to (29), we find that $\overset{(1)}{\omega}_1 = \tilde{\omega}_{ab} \eta_2^a \eta_2^b$, where the coefficients $\tilde{\omega}_{ab} = -\tilde{\omega}_{ba}$ pertain to $H_1\left(\delta|\tilde{\mathrm{d}}\right)$, i.e.,

$$\delta \tilde{\omega}_{ab} = \partial_i n_{ab}^i. \tag{33}$$

On the other hand, the general representative of $H_1\left(\delta|\tilde{\mathrm{d}}\right)$ is $\lambda = \lambda^a \mathcal{P}_{2a}$ (see (22)), with constant λ^a. Then, we can write that $\tilde{\omega}_{ab} = (1/2) f^c{}_{ab} \mathcal{P}_{2c}$, where $f^c{}_{ab} = -f^c{}_{ba}$ are constant, hence

$$\overset{(1)}{\omega}_1 = \frac{1}{2} f^c{}_{ab} \mathcal{P}_{2c} \eta_2^a \eta_2^b. \tag{34}$$

It follows that the solution to the equation associated with the antighost number equal to zero, $\delta \overset{(1)}{\omega}_1 + \gamma \overset{(0)}{\omega}_1 = \partial_i q^i$, reads

$$\overset{(0)}{\omega}_1 = \left(f^c{}_{ab} \pi_c^i A_i^b + b_a\left(y^{\alpha_0}, \partial_i y^{\alpha_0}\right)\right) \eta_2^a, \tag{35}$$

where the bosonic functions $b_a(y^{\alpha_0}, \partial_i y^{\alpha_0})$ are arbitrary at this stage. Combining the above results, we obtain the first-order deformation of the BRST charge in the form

$$\Omega_1 = \int \mathrm{d}^{D-1}x \left(\left(f^c{}_{ab}\pi^i_c A^b_i + b_a(y^{\alpha_0}, \partial_i y^{\alpha_0}) \right) \eta^a_2 + \frac{1}{2} f^c{}_{ab}\mathcal{P}_{2c}\eta^a_2\eta^b_2 \right). \tag{36}$$

Next, we investigate its second-order deformation. By direct computation we get

$$[\Omega_1, \Omega_1] = f^d{}_{[ab}f^e{}_{c]d} \int \mathrm{d}^{D-1}x \left(\eta^a_2\eta^b_2\pi^i_e A^c_i - \frac{1}{3}\eta^a_2\eta^b_2\eta^c_2\mathcal{P}_{2e} \right) + \int \mathrm{d}^{D-1}x\, \mathrm{d}^{D-1}x' \left([b_a(x), b_b(x')] - f^c{}_{ab}b_c(x)\,\delta^{D-1}(x-x') \right) \eta^a_2(x)\,\eta^b_2(x'). \tag{37}$$

Equation (5) entails the demand that $[\Omega_1, \Omega_1]$ is s -exact. However, from (37) we observe that this requirement cannot be fulfilled, so $[\Omega_1, \Omega_1]$ should vanish. This holds if and only if the following conditions are simultaneously satisfied:

$$f^d{}_{[ab}f^e{}_{c]d} = 0, \tag{38}$$

$$[b_a(x), b_b(x')] = f^c{}_{ab}b_c(x)\,\delta^{D-1}(x-x'). \tag{39}$$

The formula (38) shows that the antisymmetric constants $f^c{}_{ab}$ satisfy Jacobi's identity, being thus the structure constants of a Lie algebra. Formula (39) restricts the form of the functions b_a in the sense that they form a Lie algebra in the Poisson (Dirac) bracket, with structure constants $f^c{}_{ab}$. Due to the fact that $[\Omega_1, \Omega_1] = 0$, we deduce that we can take $\Omega_2 = \Omega_3 = \cdots = 0$. Now, we solve (39). They possess solutions if and only if the fields y^{α_0} split into two subsets

$$y^{\alpha_0} = (y^{\alpha_1}, z_{\alpha_1}), \tag{40}$$

with the properties

$$\left[y^{\alpha_1}, y^{\beta_1}\right] = 0, \quad [z_{\alpha_1}, z_{\beta_1}] = 0, \quad [y^{\alpha_1}, z_{\beta_1}] = \sigma^{\alpha_1}{}_{\beta_1}, \tag{41}$$

for some invertible matrices $\sigma^{\alpha_1}{}_{\beta_1}$ (the distributional character has been again omitted). Under these circumstances, we find that

$$b_a = z_{\alpha_1}T^{\alpha_1}_a{}_{\beta_1}\bar{\sigma}^{\beta_1}{}_{\rho_1}y^{\rho_1}, \tag{42}$$

with $\bar{\sigma}^{\beta_1}{}_{\rho_1}$ the inverse of $\sigma^{\alpha_1}{}_{\beta_1}$, and $T^{\alpha_1}_a{}_{\beta_1}$ some constant matrices, subject to the conditions

$$T^{\alpha_1}_a{}_{\beta_1}T^{\beta_1}_b{}_{\rho_1} - T^{\alpha_1}_b{}_{\beta_1}T^{\beta_1}_a{}_{\rho_1} = f^c{}_{ab}T^{\alpha_1}_c{}_{\rho_1}. \tag{43}$$

The presence of $\bar{\sigma}^{\beta_1}{}_{\rho_1}$ in (42) may in principle lead to the loss of locality. As we restrict ourselves to local deformations only, we consider the case of constant $\sigma^{\alpha_1}{}_{\beta_1}$. Therefore, the deformed BRST charge consistent to all orders in the deformation parameter is expressed by

$$\Omega = \int \mathrm{d}^{D-1}x \left(\pi^0_a\eta^a_1 - \left((\mathrm{D}_i)_a{}^b\,\pi^i_b - gz_{\alpha_1}T^{\alpha_1}_a{}_{\beta_1}\bar{\sigma}^{\beta_1}{}_{\rho_1}y^{\rho_1} \right)\eta^a_2 + \frac{1}{2}gf^c{}_{ab}\mathcal{P}_{2c}\eta^a_2\eta^b_2 \right), \tag{44}$$

with $(\mathrm{D}_i)_a{}^b = \delta_a{}^b\partial_i - gf^b{}_{ac}A^c_i$. From Ω we can gather information on the deformed constraints and modified gauge algebra. Indeed, the term in Ω linear in the ghosts η^a_2 gives rise to the deformed secondary constraints

$$\gamma_{2a} \equiv -(\mathrm{D}_i)_a{}^b\,\pi^i_b + gz_{\alpha_1}T^{\alpha_1}_a{}_{\beta_1}\bar{\sigma}^{\beta_1}{}_{\rho_1}y^{\rho_1} \approx 0, \tag{45}$$

while the term linear in the antighosts shows that these constraint functions form a Lie algebra in the Poisson (Dirac) bracket

$$[\gamma_{2a}, \gamma_{2b}] = gf^c{}_{ab}\gamma_{2c}, \tag{46}$$

with the structure constants $f^c{}_{ab}$. This completes the deformation procedure of the BRST charge for a collection of vector fields and matter fields.

5 Deformation of the BRST-invariant Hamiltonian

Further, we approach (9) and (10), etc., that control the deformation of the BRST-invariant Hamiltonian. By direct computation we find that the first term in the left hand-side of (9) reads

$$[H_{0\mathrm{B}}, \Omega_1] = -s\int \mathrm{d}^{D-1}x \times \left(f^a{}_{bc}\left(A^b_0\left(A^c_i\pi^i_a + \eta^c_2\mathcal{P}_{2a}\right) - \frac{1}{2}A^b_iA^c_jF^{ij}_a \right) + b_aA^a_0 \right) + \int \mathrm{d}^{D-1}x\ \left[\bar{H}_0, b_a\right]\eta^a_2. \tag{47}$$

We notice that the last term in the right-hand side of (47) is clearly not s-exact, so it must be compensated for by a corresponding term from $[H_1, \Omega_0]$, which can be accomplished if we take H_1 of the form

$$H_1 = \int \mathrm{d}^{D-1}x \left(f^a{}_{bc}\left(A^b_0\left(A^c_i\pi^i_a + \eta^c_2\mathcal{P}_{2a}\right) - \frac{1}{2}A^b_iA^c_jF^{ij}_a \right) + b_aA^a_0 + j \right). \tag{48}$$

The function j involves the vector fields A^a_i and the matter fields, and, moreover, we want it to fulfill the equation $\int \mathrm{d}^{D-1}x\left(\left[\bar{H}_0, b_a\right]\eta^a_2 + [j, \Omega_0]\right) = 0$, or, equivalently,

$$\left[\bar{H}_0(x_0), b_a(x)\right]\eta^a_2(x) + [j(x), \Omega_0(x_0)] = \partial_ik^i(x). \tag{49}$$

Using (14), the last equation becomes

$$\left[\bar{H}_0(x_0), b_a(x)\right]\eta^a_2(x) + \int \mathrm{d}^{D-1}x'\ \left[j(x), \pi^i_a(x')\right]\partial_i\eta^a_2(x') = \partial_ik^i(x). \tag{50}$$

In order to restrain the left hand-side to a total derivative, it is necessary that the function $j(x)$ is linear in the fields

A_i^a because the term $\left[\bar{H}_0(x_0), b_a(x)\right]$ does not involve the vector fields. Thus, we can write

$$j = j_a^i A_i^a, \tag{51}$$

where j_a^i depends only on the matter fields and their derivatives. Consequently, (50) takes the form

$$\left[\bar{H}_0(x_0), b_a(x)\right]\eta_2^a(x) + j_a^i(x)\,\partial_i\eta_2^a(x) = \partial_i k^i(x). \tag{52}$$

The left hand-side of (52) reduces to a total derivative if and only if

$$\left[\bar{H}_0(x_0), b_a(x)\right] = \partial_i j_a^i(x). \tag{53}$$

By means of the Hamilton equations with respect to the matter fields ($\partial_0 F(x) = \left[F(x), \bar{H}_0(x_0)\right]$), from (53) we derive that

$$\partial_0 b_a(x) + \partial_i j_a^i(x) = 0, \tag{54}$$

which expresses nothing but the conservation of the Hamiltonian currents[1] $j_a^\mu = \left(b_a, j_a^i\right)$. In conclusion, the consistency of the first-order deformation of the BRST-invariant Hamiltonian requires that the matter theory displays some conserved currents, equal in number with the number of vector fields A_μ^a. In the following we assume that this requirement is fulfilled. Then, the first-order deformation of the BRST-invariant Hamiltonian is given by

$$H_1 = \int \mathrm{d}^{D-1}x \left(f^a{}_{bc}\left(A_0^b\left(A_i^c\pi_a^i + \eta_2^c\mathcal{P}_{2a}\right) - \frac{1}{2}A_i^bA_j^cF_a^{ij}\right) + j_a^\mu A_\mu^a \right), \tag{55}$$

where j_a^μ stand for the conserved Hamiltonian currents mentioned in the above.

Now, we pass at the second-order deformation, described by (10). Making use of the formulas (44) and (55), we find that

$$[H_1, \Omega_1] = s\left(\int \mathrm{d}^{D-1}x\,\frac{1}{4}f^a{}_{bc}f^c{}_{de}A^{ib}A_a^jA_i^dA_j^e\right) + \int \mathrm{d}^{D-1}x\left(\left[j_b^i, b_a\right] + f^c{}_{ab}j_c^i\right)A_i^b\eta_2^a. \tag{56}$$

Looking at the form of (56), we remark that two important cases appear.

(a) If the currents j_a^i transform under the deformed gauge transformations (generated by the deformed first-class constraints)[2] according to the adjoint representation of the Lie gauge algebra

$$\left[j_b^i, b_a\right] + f^c{}_{ab}j_c^i = 0, \tag{57}$$

then the second-order deformation of the BRST-invariant Hamiltonian will be

$$H_2 = -\frac{1}{4}\int \mathrm{d}^{D-1}x\; f^a{}_{bc}f^c{}_{de}A^{ib}A_a^jA_i^dA_j^e. \tag{58}$$

As $\Omega_2 = 0$ and $[H_2, \Omega_1] = 0$, the third-order deformation equation is satisfied with the choice $H_3 = 0$. The equations for higher-order deformations will then be checked for

$$H_4 = H_5 = \cdots = 0. \tag{59}$$

(b) In the opposite situation

$$\left[j_b^i, b_a\right] + f^c{}_{ab}j_c^i \neq 0, \tag{60}$$

there appear non-trivial higher-order deformations that imply interactions among vector fields and matter fields.

In both cases, the deformed BRST-invariant Hamiltonian has the general form

$$H_{\mathrm{B}} = \int \mathrm{d}^{D-1}x\left(\frac{1}{2}\pi_{ia}\pi_i^a + \frac{1}{4}\tilde{F}_{ij}^a\tilde{F}_a^{ij} - A_0^a\left(\mathrm{D}_i\right)_a{}^b\pi_b^i + \bar{h}_0\left(y^{\alpha_0}, \partial_i y^{\alpha_0}\right) + gj_a^\mu A_\mu^a + \left(\eta_1^a - gf^a{}_{bc}\eta_2^bA_0^c\right)\mathcal{P}_{2a}\right) + O\left(g^2\right), \tag{61}$$

where

$$\tilde{F}_{ij}^a = F_{ij}^a - gf^a{}_{bc}A_i^bA_j^c, \tag{62}$$

and $O\left(g^2\right)$ is due only to the supplementary terms present in case (b).

The deformation treatment developed so far can be synthesized in three general results as follows. First, the interaction terms involving only the vector fields generate the Hamiltonian version of Yang–Mills theory, and the first-order couplings between the vector fields and matter fields is of the type $j_a^\mu A_\mu^a$, where $j_a^\mu = \left(b_a, j_a^i\right)$ are the conserved Hamiltonian currents corresponding to the matter fields. Second, the secondary first-class constraints are deformed with respect to the initial ones, and, as a consequence, the corresponding gauge transformations will be modified. Third, the deformed gauge algebra of first-class constraints is a Lie algebra. Finally, a word of caution. Once the deformations related to a given matter theory are computed, special attention should be paid to the elimination of non-locality, as well as the triviality of the resulting deformations. This completes our general procedure.

6 Applications

In the sequel we apply the general deformation procedure investigated so far to two cases of interest, where the matter theory involves scalar fields, respectively, Dirac fields.

[1] By Hamiltonian currents we understand a set of functions $j_a^\mu = \left(b_a, j_a^i\right)$ that satisfy (54) when the Hamiltonian equations of motion hold. In general, these currents do not display a manifestly covariant form. However, if we express these currents only in terms of the fields (via the elimination of the momenta on their equations of motion), we infer precisely their Lagrangian form, which is clearly covariant

[2] As j_b^i depend only on the matter fields and their derivatives, we find that their deformed gauge transformations are indeed $\bar{\delta}_\epsilon j_b^i = \left[j_b^i, \gamma_{2a}\right]\epsilon_2^a = g\left[j_b^i, b_a\right]\epsilon_2^a$

6.1 Vector fields coupled to scalar fields

First, we analyse the consistent interactions that can be introduced among a set of real scalar fields and a collection of vector fields. In this case the "free" Lagrangian action is given by

$$\tilde{S}_0^{\mathrm{L}}\left[A_\mu^a,\varphi^A\right]=\int \mathrm{d}^4x\left(-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}+\frac{1}{2}K_{AB}\left(\partial_\mu\varphi^A\right)\right.$$
$$\left.\times\left(\partial^\mu\varphi^B\right)-V\left(\varphi^A\right)\right), \tag{63}$$

where K_{AB} is an invertible symmetric constant matrix. The action (63) is invariant under the gauge transformations

$$\delta_\epsilon A_\mu^a=\partial_\mu\epsilon^a,\quad \delta_\epsilon\varphi^A=0, \tag{64}$$

and the first-class Hamiltonian is expressed by (13), with

$$\bar{h}_0=\frac{1}{2}K^{AB}\Pi_A\Pi_B-\frac{1}{2}K_{AB}\left(\partial_j\varphi^A\right)\left(\partial^j\varphi^B\right)+V\left(\varphi^A\right), \tag{65}$$

the matrix K^{AB} denoting the inverse of K_{AB}, and Π_A meaning the canonical momenta conjugated to φ^A. In this situation we have

$$y^{\alpha_0}=\left(\varphi^A,\Pi_A\right), \tag{66}$$

with

$$\left[\varphi^A,\varphi^B\right]=0,\quad \left[\Pi_A,\Pi_B\right]=0,\quad \left[\varphi^A,\Pi_B\right]=\delta^A{}_B. \tag{67}$$

By performing the identifications

$$y^{\alpha_1}\longleftrightarrow\varphi^A,\quad z_{\alpha_1}\longleftrightarrow\Pi_A,\quad \sigma^{\alpha_1}{}_{\beta_1}\longleftrightarrow\delta^A{}_B, \tag{68}$$

from (42) we obtain that the conserved charge b_a is precisely given by

$$b_a=\Pi_A T_a{}^A{}_B\varphi^B. \tag{69}$$

With b_a at hand, we then deduce the form of the currents j_a^i by employing the formula (53). Due to (65), we get

$$\left[b_a,\bar{H}_0\right]=T_a{}^A{}_B K^{BC}\Pi_A\Pi_C-\frac{\partial V}{\partial\varphi^A}T_a{}^A{}_B\varphi^B$$
$$-K_{AC}T_a{}^A{}_B\left(\partial_i\partial^i\varphi^C\right)\varphi^B. \tag{70}$$

In order to reveal some conserved Hamiltonian currents in the matter sector, it is necessary that

$$\frac{\partial V}{\partial\varphi^A}T_a{}^A{}_B\varphi^B=0, \tag{71}$$

$$\tilde{T}_{aBC}=-\tilde{T}_{aCB},\quad \bar{T}_a^{AC}=-\bar{T}_a^{CA}, \tag{72}$$

where $\tilde{T}_{aBC}\equiv K_{AB}T_a{}^A{}_C$, $\bar{T}_a^{AC}\equiv K^{BC}T_a{}^A{}_B$. Inserting (71) and (72) in (70), we arrive at

$$\left[b_a,\bar{H}_0\right]+\partial_i\left(K_{AC}T_a{}^A{}_B\left(\partial^i\varphi^C\right)\varphi^B\right)=0, \tag{73}$$

and hence the conserved currents are

$$j_a^i=K_{AC}T_a{}^A{}_B\left(\partial^i\varphi^C\right)\varphi^B. \tag{74}$$

Once we determined b_a and j_a^i, the deformed BRST charge and the first-order deformation of the BRST-invariant Hamiltonian are completely constructed. Regarding the second-order deformation of the Hamiltonian, by direct computation we deduce

$$\int \mathrm{d}^3x\ \left[j_b^i,b_a\right]A_i^b\eta_2^a=-\int \mathrm{d}^3x\ f^c{}_{ab}j_c^iA_i^b\eta_2^a$$
$$+s\left(\int \mathrm{d}^3x\ \frac{1}{2}K_{AC}T_b{}^A{}_BT_a{}^C{}_E\varphi^B\varphi^EA^{ai}A_i^b\right), \tag{75}$$

so the conditions of case (b) (see (60)) are met. Then, we infer that

$$H_2=-\int \mathrm{d}^3x\left(\frac{1}{4}f^a{}_{bc}f^c{}_{de}A^{ib}A_a^jA_i^dA_j^e\right.$$
$$\left.+\frac{1}{2}K_{AC}T_b{}^A{}_BT_a{}^C{}_E\varphi^B\varphi^EA^{ai}A_i^b\right). \tag{76}$$

As $[H_2,\Omega_1]=0$, it follows that the third-order deformation equation is verified for $H_3=0$, and similarly we can take $H_4=H_5=\cdots=0$.

Gathering the results derived so far, it follows that both the deformed BRST charge and BRST-invariant Hamiltonian can respectively be written in the form

$$\Omega=\int \mathrm{d}^3x\left(\pi_a^0\eta_1^a-\left(\left(\mathrm{D}_i\right)_a{}^b\pi_b^i-g\Pi_AT_a{}^A{}_B\varphi^B\right)\eta_2^a\right.$$
$$\left.+\frac{1}{2}gf^c{}_{ab}\mathcal{P}_{2c}\eta_2^a\eta_2^b\right), \tag{77}$$

$$H_{\mathrm{B}}=\int \mathrm{d}^3x\left(\frac{1}{2}\pi_{ia}\pi_i^a+\frac{1}{4}\tilde{F}^a_{ij}\tilde{F}_a^{ij}\right.$$
$$-A_0^a\left(\left(\mathrm{D}_i\right)_a{}^b\pi_b^i-g\Pi_AT_a{}^A{}_B\varphi^B\right)$$
$$+\frac{1}{2}K^{AB}\Pi_A\Pi_B-\frac{1}{2}K_{AB}\left(\mathrm{D}_j{}^A{}_C\varphi^C\right)\left(\mathrm{D}^{jB}{}_D\varphi^D\right)$$
$$\left.+V\left(\varphi^A\right)+\left(\eta_1^a-gf^a{}_{bc}\eta_2^bA_0^c\right)\mathcal{P}_{2a}\right), \tag{78}$$

where

$$\mathrm{D}_j{}^A{}_C=\delta^A{}_C\partial_j+gT_a{}^A{}_CA_j^a. \tag{79}$$

Now, we have enough information to identify the resulting interacting theory. Only the secondary constraints are deformed

$$\gamma_{2a}\equiv-\left(\left(\mathrm{D}_i\right)_a{}^b\pi_b^i-g\Pi_AT_a{}^A{}_B\varphi^B\right)\approx0, \tag{80}$$

and they form a Lie algebra in the Poisson bracket, like in (46). The antighost number zero piece in (78)

$$\tilde{H}_0=\int \mathrm{d}^3x\left(\frac{1}{2}\pi_{ia}\pi_i^a+\frac{1}{4}\tilde{F}^a_{ij}\tilde{F}_a^{ij}\right.$$
$$-A_0^a\left(\left(\mathrm{D}_i\right)_a{}^b\pi_b^i-g\Pi_AT_a{}^A{}_B\varphi^B\right)+\frac{1}{2}K^{AB}\Pi_A\Pi_B$$
$$\left.-\frac{1}{2}K_{AB}\left(\mathrm{D}_j{}^A{}_C\varphi^C\right)\left(\mathrm{D}^{jB}{}_D\varphi^D\right)+V\left(\varphi^A\right)\right), \tag{81}$$

represents the first-class Hamiltonian of the coupled theory. The component from (78) linear in the antighosts underlines that the Poisson brackets between the constraints and the first-class Hamiltonian are modified like

$$\left[\tilde{H}_0, G_{1a}\right] = \gamma_{2a}, \quad \left[\tilde{H}_0, \gamma_{2a}\right] = -g f^c{}_{ab} A_0^b \gamma_{2c}. \tag{82}$$

In this way, we observe that we have obtained nothing but the Hamiltonian version of the theory that describes the interaction between Yang–Mills fields and a set of scalar fields.

The Lagrangian version of the interacting model can be derived in the usual manner, thus yielding the action

$$S^{\mathrm{L}}\left[A_\mu^a, \varphi^A\right] = \int \mathrm{d}^4x \Bigg(-\frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \frac{1}{2}K_{AB}\left(\mathrm{D}_\mu^A{}_C \varphi^C\right) \times \left(\mathrm{D}^{\mu B}{}_D \varphi^D\right) - V\left(\varphi^A\right)\Bigg), \tag{83}$$

invariant under the deformed gauge transformations

$$\bar{\delta}_\epsilon A_\mu^a = \left(\mathrm{D}_\mu\right)^a{}_b \epsilon^b, \quad \bar{\delta}_\epsilon \varphi^A = g T_a^A{}_B \varphi^B \epsilon^a. \tag{84}$$

The modification of the gauge transformations, as well as the appearance of new such transformations in connection with the matter fields, is essentially due to the deformation of the secondary first-class constraints like in (80). If in (83) and (84) we make the transformation $T_a^A{}_B \to \mathrm{i}T_a^A{}_B$, we derive the non-abelian analogue of scalar electrodynamics.

6.2 Vector fields coupled to Dirac fields

Finally, we examine the consistent couplings between a set of vector fields and a collection of massive Dirac fields. In view of this, we start from the Lagrangian action

$$\tilde{S}_0^{\mathrm{L}}\left[A_\mu^a, \psi^\alpha_A, \bar{\psi}_\alpha^A\right] = \int \mathrm{d}^4x \Bigg(-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar{\psi}_\alpha^A \times \left(\mathrm{i}\left(\gamma^\mu\right)^\alpha{}_\beta \partial_\mu - m\delta^\alpha{}_\beta\right)\psi^\beta_A\Bigg), \tag{85}$$

where ψ^α_A and $\bar{\psi}_\alpha^A$ denote the fermionic spinor components of the Dirac fields ψ_A and $\bar{\psi}^A$. The action (85) is invariant under the gauge transformations

$$\delta_\epsilon A_\mu^a = \partial_\mu \epsilon^a, \quad \delta_\epsilon \psi^\alpha_A = 0, \quad \delta_\epsilon \bar{\psi}_\alpha^A = 0. \tag{86}$$

The purely matter theory is subject to the second-class constraints

$$\bar{\chi}^\alpha_A \equiv \bar{\Pi}^\alpha_A + \frac{\mathrm{i}}{2}\left(\gamma^0\right)^\alpha{}_\beta \psi^\beta_A \approx 0, \qquad \chi_\alpha^A \equiv \Pi_\alpha^A + \frac{\mathrm{i}}{2}\left(\gamma^0\right)^\beta{}_\alpha \bar{\psi}_\beta^A \approx 0, \tag{87}$$

where Π_α^A and $\bar{\Pi}^\alpha_A$ represent the canonical momenta respectively conjugated to ψ^α_A and $\bar{\psi}_\alpha^A$. By eliminating the constraints (87) with the help of the Dirac bracket, we find for the model under consideration that

$$y^{\alpha_0} = \left(\psi^\alpha_A, \bar{\psi}_\alpha^A\right), \tag{88}$$

where the fundamental Dirac brackets are expressed by

$$\left[\psi^\alpha_A, \psi^\beta_B\right] = 0, \quad \left[\bar{\psi}_\alpha^A, \bar{\psi}_\beta^B\right] = 0, \qquad \left[\psi^\alpha_A, \bar{\psi}_\beta^B\right] = -\mathrm{i}\left(\gamma^0\right)^\alpha{}_\beta \delta_A{}^B. \tag{89}$$

The first-class Hamiltonian is of the type (13), with

$$\bar{h}_0 = -\bar{\psi}_\alpha^A\left(\mathrm{i}\left(\gamma^i\right)^\alpha{}_\beta \partial_i - m\delta^\alpha{}_\beta\right)\psi^\beta_A. \tag{90}$$

If we make the identifications

$$y^{\alpha_1} \longleftrightarrow \psi^\alpha_A, \quad z_{\alpha_1} \longleftrightarrow \bar{\psi}_\alpha^A, \quad \sigma^{\alpha_1}{}_{\beta_1} \longleftrightarrow -\mathrm{i}\left(\gamma^0\right)^\alpha{}_\beta \delta_A{}^B, \tag{91}$$

from (42) we deduce that the conserved charge b_a reads

$$b_a = \mathrm{i}\bar{\psi}_\alpha^B\left(\gamma^0\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A. \tag{92}$$

On account of (90) we derive that

$$\left[b_a, \bar{H}_0\right] = -\partial_i\left(\mathrm{i}\bar{\psi}_\alpha^B\left(\gamma^i\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A\right), \tag{93}$$

hence the conserved currents will be

$$j_a^i = \mathrm{i}\bar{\psi}_\alpha^B\left(\gamma^i\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A. \tag{94}$$

In order to fully determine the interacting theory, it remains to analyse the higher-order deformations of the BRST-invariant Hamiltonian. Direct computation yields

$$\left[j_b^i, b_a\right] = -f^c{}_{ab} j_c^i, \tag{95}$$

such that we are in case (a) (see (57)). Consequently, we find that H_2 is like in (58), and also $H_3 = H_4 = \cdots = 0$.

Putting together the results obtained until now, it results that the deformed BRST charge and new BRST-invariant Hamiltonian can be written

$$\Omega = \int \mathrm{d}^3x \Bigg(\pi_a^0 \eta_1^a - \left(\left(\mathrm{D}_i\right)_a{}^b \pi_b^i - \mathrm{i}g\bar{\psi}_\alpha^B\left(\gamma^0\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A\right) \times \eta_2^a + \frac{1}{2} g f^c{}_{ab}\mathcal{P}_{2c}\eta_2^a\eta_2^b\Bigg), \tag{96}$$

$$H_{\mathrm{B}} = \int \mathrm{d}^3x \Bigg(\bar{\psi}_\alpha^B\left(\mathrm{i}\left(\gamma^i\right)^\alpha{}_\beta \mathrm{D}_i^A{}_B - m\delta^\alpha{}_\beta \delta^A{}_B\right)\psi^\beta_A + \frac{1}{2}\pi_{ia}\pi_i^a + \frac{1}{4}\tilde{F}^a_{ij}\tilde{F}^{ij}_a - A_0^a\left(\left(\mathrm{D}_i\right)_a{}^b \pi_b^i - \mathrm{i}g\bar{\psi}_\alpha^B\left(\gamma^0\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A\right) + \left(\eta_1^a - g f^a{}_{bc}\eta_2^b A_0^c\right)\mathcal{P}_{2a}\Bigg). \tag{97}$$

From the analysis of the above quantities, we see that the modified constraints are the secondary ones, namely,

$$\gamma_{2a} \equiv -\left(\left(\mathrm{D}_i\right)_a{}^b \pi_b^i - \mathrm{i}g\bar{\psi}_\alpha^B\left(\gamma^0\right)^\alpha{}_\beta T_a^A{}_B \psi^\beta_A\right) \approx 0, \tag{98}$$

and they form again a Lie algebra in terms of the Dirac bracket, just like in (46). The antighost number zero piece from (97)

$$\tilde{H}_0 = \int \mathrm{d}^3x \Bigg(\bar{\psi}_\alpha^{\ B} \left(\mathrm{i} \left(\gamma^i\right)^\alpha_{\ \beta} \mathrm{D}_{i\ B}^{A} - m\delta^\alpha_{\ \beta}\delta^A_{\ B} \right) \psi^\beta_{\ A} + \frac{1}{2}\pi_{ia}\pi_i^a + \frac{1}{4}\tilde{F}^a_{ij}\tilde{F}^{ij}_a - A^a_0 \left(\left(\mathrm{D}_i\right)_a^{\ b} \pi^i_b - \mathrm{i}g\bar{\psi}_\alpha^{\ B} \left(\gamma^0\right)^\alpha_{\ \beta} T^A_{a\ B}\psi^\beta_{\ A} \right) \Bigg), \quad (99)$$

gives the deformed first-class Hamiltonian, while the term from (97) linear in the antighosts indicates that the Dirac brackets between the constraints and the first-class Hamiltonian change like in (82). In conclusion, we are led to the Hamiltonian formulation of the model describing the interaction between Yang–Mills fields and a collection of spinor fields.

The Lagrangian setting of this interacting model is described by the action

$$S^{\mathrm{L}} \left[A^a_\mu, \varphi^A\right] = \int \mathrm{d}^4x \Bigg(-\frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \bar{\psi}_\alpha^{\ B} \left(\mathrm{i} \left(\gamma^\mu\right)^\alpha_{\ \beta} \mathrm{D}^A_{\mu\ B} - m\delta^\alpha_{\ \beta}\delta^A_{\ B} \right) \psi^\beta_{\ A} \Bigg), \quad (100)$$

subject to the deformed gauge transformations

$$\bar{\delta}_\epsilon A^a_\mu = \left(\mathrm{D}_\mu\right)^a_{\ b}\epsilon^b, \quad \bar{\delta}_\epsilon \psi^\alpha_{\ A} = gT^B_{a\ A}\psi^\alpha_{\ B}\epsilon^a, \quad \bar{\delta}_\epsilon \bar{\psi}_\alpha^{\ A} = -gT^A_{a\ B}\bar{\psi}_\alpha^{\ B}\epsilon^a, \quad (101)$$

which are again a consequence of the new constraints (98). Like in the scalar case, if in (100) and (101) we perform the replacement $T^A_{a\ B} \to \mathrm{i}T^A_{a\ B}$ and consider an $SU(3)$ gauge algebra, we arrive at quantum chromodynamics.

7 Conclusion

In conclusion, in this paper we have investigated the consistent Hamiltonian interactions that can be introduced between a set of vector fields and a system of matter fields by using some cohomological techniques. This problem involves two steps. Initially, we deform the "free" BRST charge, and we subsequently approach the deformation of the BRST-invariant Hamiltonian. Related to the BRST charge, we notice that only the first-order deformation is non-trivial, while its consistency requires the deformed first-class constraints form a Lie algebra. Regarding the BRST-invariant Hamiltonian, we have shown that the first-order interaction contains two terms. The first one describes an interaction among the vector fields. The second term appears only if the matter theory possesses some conserved Hamiltonian currents and is of the form $j^\mu_a A^a_\mu$, where j^μ_a are the currents. The second-order deformation of the BRST-invariant Hamiltonian contains the spatial part of the quartic vertex of pure Yang–Mills theory. If the currents j^i_a transform under the deformed gauge transformations according to the adjoint representation of the Lie gauge algebra, then all the other deformations involving the matter fields, of order two and higher, vanish. In the opposite case, at least the second-order deformation implying matter fields is non-vanishing, but in principle there might be other non-trivial terms. The general procedure has been applied to the study of the interactions between a set of vector fields and a collection of real scalar fields, respectively, a set of Dirac fields.

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